MULTIMEDIA UNIVERSITY

**STUDENT ID NO**

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 1, 2018/2019**

## DIM5068 – MATHEMATICAL TECHNIQUES 2

(For DIT students only)

25 OCTOBER 2018
2.30 p.m. – 4.30 p.m.
(2 Hours)

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**INSTRUCTIONS TO STUDENT**

1. This question paper consists of **2** pages with **4** questions excluded the cover page and Appendix. Key formulae are given in the Appendix.

2. Answer **ALL** questions.

3. Write your answers in the answer booklet provided.

4. All necessary working steps must be shown.

**Question 1**

a. Differentiate the following functions with respect to $x$ by using **Chain Rule.**

i) $f(x) = \ln\left(2x^3 - 4x^2 - 4x\right)$. (5 marks)

ii) $y = -\dfrac{6}{\sqrt[3]{2x^3 + 4x}}$. (6 marks)

b. If $7y^2 - 4x^5 + 2xy^2 = x^4 y$, show that $\dfrac{dy}{dx} = \dfrac{20x^4 - 2y^2 + 4x^3 y}{14y + 4xy - x^4}$. (6 marks)

c. Given $f(x) = x^3 - 6x^2$

i) Find the intervals on which the function is increasing and decreasing. (6 marks)

ii) Identify the function's local extreme values. (2 marks)

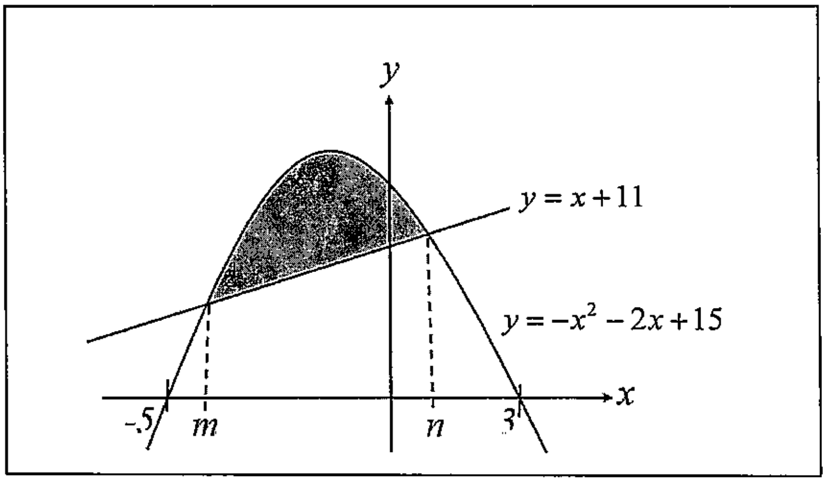
**[TOTAL 25 MARKS]**

**Question 2**

a. Use **Substitution Rule** to find $\int \left(3x^3 - 1\right)\left(3x^4 - 4x + 11\right)^{10} dx$. (7 marks)

b. Determine $\int (2x-1)e^{2x-1} dx$ by using the **Integration by Parts.** (7 marks)

c. The diagram below shows the curve of $y = -x^2 - 2x + 15$ and the straight line of $y = x + 11$.




i) Show that the value of $m = -4$ and $n = 1$. (5 marks)

ii) Find the area of the shaded region. (6 marks)

**[TOTAL 25 MARKS]**

**Continued...**

**Question 3**

a. Solve the differential equation, $\frac{dy}{dx} = \frac{7 - 3x^2 + \sec^2 x}{y^3}$ by using **separable method.** (5 marks)

b. Use the **method of integrating factors** to solve the differential equation, $x^5 \frac{dy}{dx} + 3x^4 y = x^9 + x^2 e^x$ given that $y(0) = 100$. (11 marks)

c. Find the **general solution** of non-homogeneous equation, $y'' + 8y' - 33y = 66$ which consists of **complementary solution,** $y_c$ and **particular solution,** $y_p$. (9 marks)

**[TOTAL 25 MARKS]**

**Question 4**

a. Let $\mathbf{a} = 4\mathbf{i} - 2\mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

i) Compute $2\mathbf{b} \bullet (-3\mathbf{a})$. (4 marks)

ii) Find the value of $x$ and $y$ if $\mathbf{a} + \mathbf{b} = \langle y + 3x, x, 1 \rangle$. (3 marks)

b. Andy wants to sketch a triangular shape. Given the vertices of the triangle $A = (2, 1, 0)$, $B = (3,5,7)$, and $C = (4, 3, 10)$.

i) Determine $\vec{AB}$ and $\vec{AC}$. (2 marks)

ii) Calculate the cross product of $\vec{AB}$ and $\vec{AC}$. (3 marks)

iii) Compute the total area of the triangle. Round your answer to 2 decimal places. (3 marks)

c. If a line passing through the points (-2, 1, -6) and (0, 4, -2), compute the

i) parametric equations of the line. (4 marks)

ii) symmetric equations of the line. (3 marks)

d. Find an **equation of the plane** that passes through the point (3,8,-5) and is perpendicular to $5\vec{i} + 4\vec{j} - 6\vec{k}$.
(3 marks)

**[TOTAL 25 MARKS]**

**End of page.**

# APPENDIX

**Derivatives:** $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

## Differentiation Rules

### *General Formulae*

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}(x^n)=nx^{n-1}$
4. $\dfrac{d}{dx}[f(u)]=\dfrac{dy}{du}.\dfrac{du}{dx}$

### *Exponential and Logarithmic Functions*

1. $\dfrac{d}{dx}(e^x)=e^x$
2. $\dfrac{d}{dx}(a^x)=a^x\ln a$
3. $\dfrac{d}{dx}(\ln x)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\ln a}$

### *Trigonometric Functions*

1. $\dfrac{d}{dx}(\sin x)=\cos x$
2. $\dfrac{d}{dx}(\cos x)=-\sin x$
3. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
4. $\dfrac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
6. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$

## Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\dfrac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int \dfrac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int \sin u\,du=-\cos u+C$
6. $\int \cos u\,du=\sin u+C$
7. $\int \sec^2 u\,du=\tan u+C$
8. $\int \csc^2 u\,du=-\cot u+C$
9. $\int \sec u\tan u\,du=\sec u+C$
10. $\int \csc u\cot u\,du=-\csc u+C$

## Application of Integrals:

***Areas between Curve,*** $A=\int_a^b[f(x)-g(x)]\,dx$

## Differential Equations

***Linear Differential Equations***

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\, dx, \text{ where } \mu = e^{\int p(x)\,dx}$$

***Constant Coefficient of Homogeneous Equations***

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

***General Solutions to the Auxiliary Equation:***

| | |
|---|---|
| 2 Real & Unequal Roots ($b^2 - 4ac > 0$) | $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$ |
| Repeated Roots ($b^2 - 4ac = 0$) | $y = c_1 e^{rx} + c_2 x e^{rx}$ |
| 2 Complex Roots ($b^2 - 4ac < 0$) | $y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$ |

***Constant Coefficient of Non-Homogeneous Equations***

$y = y_c + y_p$ [$y_c$: complementary solution, $y_p$: particular solution]

## Vector

***Length of Vector***

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

***Dot Product***

If $\theta$ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos \theta$

***Cross Product***

If $\theta$ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$

**Area for parallelogram *PQRS***

$= \left|\vec{PQ} \times \vec{PR}\right|$

**Area for triangle PQR**

$= \frac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|$

***Equation of Lines***

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

***Equation of Planes***

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

***Angle between Two Planes:*** $\theta = \cos^{-1}\left(\dfrac{\mathrm{n}_1 \bullet \mathrm{n}_2}{|\mathrm{n}_1||\mathrm{n}_2|}\right)$